Plenary Talks

Henrique Leitão (Universidade de Lisboa)

Monday 30 June, 09:00–09:45 • Auditorium

Plenary Session I

Alentejo in Science: Scientific Discussions at the Badajoz-Elvas Border in 1524

TBA

Robert Calderbank (Duke University)

Monday 30 June, 09:45–10:30 • Auditorium

Plenary Session I

Teaching Old Sequences New Tricks

Periodic sequences with good correlation properties have found many applications in communications and radar systems. Many families of sequences are constructed from quadratic forms over the binary field, an area to which Peter Cameron has made many contributions. Emerging applications to 6G wireless systems require good correlation properties in both delay and Doppler, and this talk will describe how familiar families of sequences are finding new applications.

Pablo Spiga (University of Milano-Bicocca)

Monday 30 June, 16:30–17:15 • Auditorium

Plenary Session II

Kronecker classes, Isbell's conjecture and derangement graphs

A derangement is a permutation with no fixed points — a simple concept with implications in group theory and beyond. A normal covering of a finite group G is a collection of proper subgroups $H_1, \ldots, H_\ell$ such that every element of G lies in a conjugate of one of the H_i. In this talk, we explore some interplays between derangements, normal coverings, and Kronecker classes in algebraic number fields. Along the way, we present a series of open questions and show how problems in permutation group theory, derangement graphs, and algebraic number theory are connected.

Mikhail V. Volkov ( )

Tuesday 1 July, 09:00–09:45 • Auditorium

Plenary Session III

Completely Reachable Automata: an interplay between (semi)groups, finite automata, and binary trees

A finite automaton is called completely reachable if every non-empty set of states arises the image of a certain sequence of input signals. Complete reachability is a strengthening of the well-known property of synchronizability. We overview recent results on completely reachable automata, with an emphasis on connections to the theory of permutation groups.

Gordon Royle (University of Western Australia)

Tuesday 1 July, 09:45–10:30 • Auditorium

Plenary Session III

Cubic graphs and spectral gap sets

Spectral graph theory is the study of the relationship between the graphical properties of a graph and the spectral properties (i.e., eigenvalues and eigenvectors) of one of the various matrices associated with that graph, most commonly the adjacency matrix. The spectrum of the adjacency matrix of a cubic graph (i.e., one where each vertex has three neighbours) on n vertices is a set of n real numbers lying in the interval [-3,3] and it determines a surprising amount of information about the graph. A spectral gap set is an open subset X of (-3,3) with the property that there are an infinite number of cubic graphs whose spectrum is disjoint from X. For example, the interval (-3,-2) is a spectral gap set because the infinite family of cubic line graphs has no eigenvalues in (-3,-2), and in fact the precise list of all cubic graphs whose spectrum avoids (-3,-2) is known. Krystal Guo and Bojan Mohar showed that the interval (-1,1) is a spectral gap set for cubic graphs, and recently Alicia Kollár and Peter Sarnak demonstrated the same result for (-2,0) and in addition showed that any spectral gap interval has length at most 2. In this talk

I describe some recent work, joint with Krystal Guo, where we give exact characterisations of the cubic graphs with spectra avoiding (-1,1) and those with spectra avoiding (-2,0). These exact characterisations allow us to deduce that (-1,1) is a maximal spectral gap set, thereby answering a question of Kollár and Sarnak. The talk is largely non-technical and should be accessible to anyone familiar with basic graph theory and linear algebra.

Nik Ruskuc (University of St Andrews)

Tuesday 1 July, 16:30–17:15 • Auditorium

Plenary Session IV

Heights of congruence lattices of semigroups

The height of a (finite) lattice L is the size of a maximum chain in L. Cameron, Solomon and Turull (1989) showed that the height of the subgroup lattice of the symmetric group S_n is given by $\lceil 3n/2\rceil - b(n)$, where b(n) is the number of 1s in the binary expansion of n. The analogue of S_n for semigroups is the full transformation semigroup T_n. Cameron, Gadoleau, Mitchell and Perese (2017) established an accurate asymptotic formula for the height of the subsemigroup lattice of T_n. But the subgroup lattice of a group G can be viewed from a different angle: it is (isomorphic to) the lattice of right (or left) congruences of G.(And one-sided congruences of a monoid are in a 1-1 correspondence with cyclic transformation representations of S.) In this talk I will introduce a general method which gives a lower bound for the height of the lattices of one- or two-sided congruences of an arbitrary semigroup, and under certain additional conditions gives the exact values. I will apply this theory to obtain the height for the lattices of right and left congruences of T_n, as well as for many other natural semigroups of transformations, partitions and matrices. This is joint work with Matthew Brookes, James East, Craig Miller and James Mitchell.

Martin Liebeck (Imperial College London)

Wednesday 2 July, 09:00–09:45 • Auditorium

Plenary Session V

Fixed point spaces for actions of finite and algebraic groups

I will present some old and new results giving bounds for the sizes of fixed point spaces of elements of finite and algebraic groups G in their actions on various G-sets, G-modules and G-varieties. A guiding motivation is an old conjecture of Peter Neumann that every non-regular primitive permutation group of degree n has an element g such that fix(g) is at least 1 and at most the square root of n.

Eamonn O'Brien (University of Auckland)

Wednesday 2 July, 09:45–10:30 • Auditorium

Plenary Session V

Challenging problems for matrix groups

Significant progress has been achieved in developing high-quality algorithms to answer questions about matrix groups defined over finite fields. But major challenges remain. I will report on the progress and discuss some of these difficult problems.

Maria Elisa Fernandes (Universidade de Aveiro)

Wednesday 2 July, 11:00–11:45 • Auditorium

Plenary Session VI

Independent sets and geometries for symmetric and alternating groups

An *independent set* in a group G is a subset of elements such that no element lies in the subgroup generated by the others. The problem of determining the maximal size $\mu(G)$ of such a set gained relevance following results by Diaconis and Saloff-Coste giving the bound $|G|^{O(\mu(G))}n^2\log(n)$ for the time it takes for a random walk on the group G to approach the uniform distribution. When G is the symmetric group $\mathrm{Sym}(n)$, Whiston proved that the maximal size of an independent generating set is $n-1$. Cameron and Cara later classified all such maximal independent sets and demonstrated that they are *strongly independent*, meaning they satisfy the *intersection property (IP)*: $\langle \rho_i : i \in J\rangle \cap \langle \rho_i : i \in K\rangle = \langle \rho_i : i \in J\cap K\rangle$, for any subsets J, K of the index set.

This property plays a central role in Tits's theory of coset geometries. In particular, abstract regular polytopes can be constructed from strongly independent sets of involutions. Moreover, these sets can be ordered such that non-consecutive generators commute — a property known as the *string property (SP)*.

A group generated by involutions is called a *C-group*, a *sggi*, or a *string C-group* if it satisfies the intersection property (IP), the string property (SP), or both, respectively. Coset geometries arising from C-groups exhibit many structural similarities with abstract regular polytopes and are known as *regular hypertopes*. The classification of Cameron and Cara of maximal independent sets for $\mathrm{Sym}(n)$ includes the classification of regular hypertopes of maximal rank. Only one set of generators in this classification is a string C-group, namely $[(1\,2), (2\,3), \ldots, (n{-}1\,n)]$, which is the automorphism group of the regular simplex.

For the alternating group $\mathrm{Alt}(n)$, it is known that the maximal rank of a string C-group (equivalently, the rank of an abstract regular polytope with automorphism group $\mathrm{Alt}(n)$) is $\lfloor (n-1)/2 \rfloor$ when $n \geq 12$.

More recently, we have shown that the maximal size of an independent sggi in $\mathrm{Alt}(n)$ is $\lfloor \frac{3(n-1)}{5} \rfloor$ for $n \geq 9$, and showed that this bound is tight when $n \equiv 0, 1, 4 \pmod 5$.

This talk will present these results and explore several related resolved and open problems. The work discussed here was carried out in collaboration with Jessica Anzanello, Peter Brooksbank, Peter Cameron, Dimitri Leemans, Mark Mixer, and Pablo Spiga.

Cheryl E Praeger (University of Western Australia)

Wednesday 2 July, 14:00–14:45 • Auditorium

Plenary Session VII

(The Fun of) Working with Peter

I am one of many who have enjoyed the fun of working with Peter Cameron. And I am Super-happy to have this opportunity to wander through the story of our mathematical collaborations. Although Peter and I were born in the same town in South East Queensland, I did not meet Peter until I reached Oxford as a DPhil student. Our first joint maths project dates back to the announcement finite simple group classification, and working jointly has continued for 45 years. Speaking with Peter leads to "all kinds of new ideas".

Colva Roney-Dougal (University of St Andrews)

Wednesday 2 July, 14:50–15:35 • Auditorium

Plenary Session VII

Bases and relational complexity

Let G be a permutation group on a set Ω. An *irredundant base* for G is a sequence $(\omega_1, \ldots, \omega_b)$ of elements of Ω such that the corresponding sequence of point stabilisers satisfies

$$G > G_{\omega_1} > G_{\omega_1,\omega_2} > \cdots > G_{\omega_1,\ldots,\omega_b} = 1.$$

There is an unexpected connection to model theory, via the notion of *relational complexity*. Informally speaking, the relational complexity of G measures the extent to which local information about the action of elements of G determines global information. This talk will be a survey of these two areas, both of which have a very Cameron-esque flavour.

Persi Diaconis (Stanford University)

Wednesday 2 July, 16:30–17:15 • Auditorium

Plenary Session VIII

Combinatorics and the Sylow theorems

There are simple, interesting problems that remain open about the Sylow-p subgroups of the symmetric group(!). One is to enumerate the number and sizes of the Sylow-p double cosets. A second is to understand the way double cosets multiply and combine (Hecke Algebra). In joint work with Eugenio Giannelli, Bob Guralnick, Stacey Law, Gabriel Navarro, Benjamin Sambale and Hunter Spink, we offer a few answers and a few open problems. I'll try to explain all this in an elementary way.

Victoria Gould (University of York)

Thursday 3 July, 09:00–09:45 • Auditorium

Plenary Session IX

Diameter of pseudo-finite semigroups

A semigroup S is said to be *right pseudo-finite* if the universal right congruence on S can be generated by a finite set $U \subseteq S \times S$, and there is a bound on the length of derivations for an arbitrary pair $(s,t) \in S \times S$

as a consequence of those in U. The *right diameter* of such a semigroup is then the smallest bound taken over all finite generating sets. There is a dual notion of being *left pseudo-finite* and of *left diameter*. The properties of being right, or left, pseudo-finite are finitary conditions, in that any finite S is both right and left pseudo-finite with right and left diameter 0 (if S is trivial) or 1 (take $U = S \times S$). On the other hand, some well-known uncountable semigroups also have right and left diameter 1.

This talk will give an introduction to the notion of pseudo-finiteness and diameter, explaining how they arise arise from a number of diverse sources. We then focus on the right and left diameter of some natural semigroups of mappings of sets, and of order-preserving mappings of chains. In the latter case, the right diameter is determined by the structure of the chain. So far, for all 'natural' right or left pseudo-finite semigroups, the corresponding diameter has been found to be at most 4.

The work presented comes from many sources: the most recent is joint work with James East, Craig Miller, Tom Quinn-Gregson and Nik Ruškuc.

Marianne Johnson (University of Manchester)

Thursday 3 July, 09:45–10:30 • Auditorium

Plenary Session IX

Forbidden configurations for coherency

Coherency is one of a suite of natural finitary conditions for monoids: a monoid S is said to be right (dually, left) coherent if every finitely generated subact of every finitely presented right (dually, left) S-act is finitely presented, and S is said to be coherent if it is both right and left coherent. The problem of determining whether a given monoid (or is not) coherent can be technically challenging, and disparate techniques have been developed to tackle this question for several interesting monoids, including the monoids arising as the free objects in each of the following varieties: monoids, groups, inverse monoids, ample monoids, and left ample monoids. We exhibit a particular configuration of elements that prohibits left (respectively, right) coherence in certain monoid subsemigroups S of a right (respectively, left) E-Ehresmann monoid F, and apply this technique to show that the free left Ehresmann monoid of rank at least 2 is not left coherent and the free Ehresmann monoid of rank at least 2 is neither left nor right coherent. Our technique simplifies significantly in the case where the overmonoid F is an E-unitary inverse monoid, and we apply this to recover the (negative) results of Gould and Hartmann, namely: the free inverse monoids and free ample monoids of rank at least 2 are neither left nor right coherent, and the free left ample monoid of rank at least 2 is not left coherent. In a positive direction, we show that the free left Ehresmann monoid is weakly coherent.

This is joint work with Victoria Gould.

Peter Cameron (University of St Andrews)

Thursday 3 July, 16:30–17:15 • Auditorium

Plenary Session X

SEMPER ABSTRACTA

One of the figures in the magnificent azulejos in the Geometry hall in the University of Evora carries a banner saying "SEMPER ABSTRACTA". This is good advice for mathematicians: the power of our subject rests partly in the fact that our abstract arguments and theorems can be applied to a wide range of concrete situations. However, the advice is a bit problematic. Most mathematicians would regard numerical computation as being discrete, but perhaps algebraic computation is more abstract. Also, graph theory is prob- ably more concrete than algebraic geometry. The borderline is often unclear. I will discuss several topics that have arisen in my own work where the shifting boundary between concrete and abstract is important. I plan to include graphs with smallest eigenvalue -2; the random graph and Zermelo–Fraenkel set theory; and two ways of looking at Frobenius groups.
