TS13: Thematic Session: Geometric and combinatorial (inverse) semigroup theory 2

Tuesday 1 July, 14:00–16:00 • Room 110

Mark Kambites (University of Manchester)

Time: 14:00–14:30

Maximal subgroups of special inverse monoids 1

I will discuss what is known, and what remains unknown, about the group of units and other subgroups of inverse monoids defined by presentations where all the relations take the form $w = 1$. The original results mentioned are joint work with Robert D. Gray (University of East Anglia).

Jan Philipp Wächter (University of Manchester)

Time: 14:30–15:00

Maximal subgroups of special inverse monoids 2

A monoid is called special if it admits a monoid presentation where all relations are of the form $w = 1$. Accordingly, the class of special monoids includes all groups. However, the connection between special monoids and groups is much deeper as they inherit many properties from their group of units. One example for this phenomenon is that the word problem of a special monoid is decidable if and only if the one of its unit group is (Makanin). This immediately yields that also the word problems of all other subgroups of the special inverse monoid are decidable. In fact, Malheiro showed that all maximal (w.r.t. set inclusion) subgroups are isomorphic to the group of units, underlining how much the structure of the monoid is dominated by its group of units.

Analogously to special monoids, we may also consider special inverse monoids (which admit an inverse monoid presentation where all relations are of the form $w = 1$). From an algorithmic perspective, special inverse monoids are interesting since the word problem for one-relator monoids (i.e. monoids admitting a presentation with a single relation $u = v$) reduces to the word problem for one-relator special inverse monoids (Ivanov, Margolis and Meakin). One would hope that the structure of special inverse monoids is similarly dominated by the group of units as in the non-inverse case. Indeed, for the class of E-unitary special inverse monoids, Gray and Kambites showed that every maximal subgroup is virtually embeddable in the group of units (i.e. has an embeddable finite index subgroup). Outside the realm of E-unitary inverse monoids, however, this is far from true: we will show a construction that yields any finitely presented group (even one with an undecidable word problem) as a maximal subgroup of a special inverse monoid with a trivial group of units. We will also discuss techniques for showing this result.

This is joint work with Robert Gray and Mark Kambites.

Nóra Szakács (University of Manchester)

Time: 15:00–15:30

Inverse semigroups with bounded group distortion

Given an inverse semigroup S, the maximal group morphism from S to S/σ induces maps from the Schützenberger graphs of S to the Cayley graph of S/σ, which are injective when the inverse semigroup is E-unitary. We consider how these embeddings distort the distance function on the Schützenberger graphs and call this the group distortion of S. We are motivated by the following questions: if the group distortion is sufficiently nice (e.g. recursively bounded, or linearly bounded, or even an isometry), does that imply a solvable word problem in S? And: what conditions can we impose on S to ensure that the group distortion is sufficiently nice? We will show that much of what one would hope for fails, but will also present some positive results. This is based on joint work with Mark Kambites.

Igor Dolinka (University of Novi Sad)

Time: 15:30–16:00

Some new results on right units of special inverse monoids

I will present the recent results – in collaboration with Robert D. Gray (University of East Anglia) – aimed at the study and possible characterisation of (right cancellative, finitely generated) monoids arising as the monoids of right invertible elements of finitely presented special inverse monoids. In particular, we compare the class $\mathcal{RU}$ of such monoids with: the class $\mathcal{P}$ of prefix monoids of finitely presented groups,

the class $\mathcal{RC}_1$ of finitely generated submonoids of finitely RC-presented monoids, and $\mathcal{RC}_2$, the class of recursively RC-presented monoids. Our conclusions follow from several general results on finite generation and presentability of maximal subgroups of special inverse monoids, stemming from some arguments with a definite geometric flavour.
