TS14: Thematic Session: Geometric and combinatorial (inverse) semigroup theory 3

Thursday 3 July, 14:00–16:00 • Room 110

Carl-Fredrik Nyberg Brodda (Korea Institute for Advanced Study)

Time: 14:00–14:30

The growth of free inverse monoids

I'll present recent results joint with Kambites, Szakács, and Webb on computing the exact exponential growth rates of non-monogenic free inverse monoids, which turns out to be a very mysterious sequence of algebraic numbers. I'll contrast this with the growth in other free algebraic structures, including that of free regular *-monoids arising in work by East, Muhammed, Gray, and Ruškuc.

Daniel Heath (University of Manchester)

Time: 14:30–15:00

Growth of monogenic free adequate monoids

The class of (left) adequate semigroups forms a quasivariety sitting above inverse semigroups, and as such free objects exist for any rank. Akin to Munn's folded tree descriptions for the free inverse semigroup, the free (left) adequate semigroup has a tree-like model due to Kambites, with retractions forming the transition from Stallings' foldings. This description is somewhat technical, but we use this description in considering the growth of $\mathrm{FLAd}(X)$ in terms of edges of trees. Whilst this appears difficult in general, we present results in the monogenic case and show that FLAd_1 grows intermediately in terms of partitions of naturals, whilst FAd_1 grows exponentially. This is joint work with T. Aird (University of Manchester).

Ajda Lemut Furlani (University of of Ljubljana)

Time: 15:00–15:30

The solution of the word problem in the free F-birestriction monoid in enriched signature

Birestriction monoids in which every σ-class has a maximum element are called F-birestriction monoids. They are non-regular analogues of F-inverse monoids and form a variety of algebras in the extended signature $(\cdot, ^*, ^+, m, 1)$, where m is the unary operation that maps each element to the maximum element of its σ-class. The talk will present the solution of the word problem for the free X-generated F-birestriction monoid $\mathsf{FFBR}(X)$ and its strong and perfect analogues. We find a decomposition of $\mathsf{FFBR}(X)$ as a partial action product of the idempotent semilattice of its universal inverse monoid by the free monoid X^* which reduces the word problem in $\mathsf{FFBR}(X)$ to the word problem in its universal inverse monoid. We solve the latter problem by applying Stephen's procedure and showing that Schützenberger graphs of elements of this inverse monoid are finite and effectively constructible. Similar methods apply to solving the word problem in the strong and perfect analogues of $\mathsf{FFBR}(X)$. This is joint work with Ganna Kudryavtseva.

James East (Western Sydney University)

Time: 15:30–16:00

Transformation representations of diagram monoids

Cayley's Theorem states that any finite monoid can be faithfully represented as a semigroup of transformations (self-maps) of a finite set. The minimum size of such a set is the *(minimum transformation) degree* of the monoid.

We obtain formulae for the degrees of the most well-studied families of finite diagram monoids, including the partition, Brauer, Temperley–Lieb and Motzkin monoids. For example, the partition monoid $\mathcal{P}_n$ has degree $1 + \frac{B(n+2)-B(n+1)+B(n)}{2}$ for $n \geq 2$, where these are Bell numbers. The proofs involve constructing explicit faithful representations of the minimum degree, many of which can be realised as (partial) actions on projections.

This is joint work with Reinis Cirpons and James Mitchell, both at Univ St Andrews.
