TS4: Thematic Session: Groups and Graphs

Monday 30 June, 11:00–12:28 • Room 103

Đorđe Mitrović (University of Auckland)

Time: 11:00–11:22

A new family of transitive permutation groups with exponential graph growth

Let Γ be a finite connected graph and G a vertex-transitive group of its automorphisms. The pair (Γ, G) is called locally-L if the permutation group induced by the action of the vertex-stabiliser G_v on the neighbourhood of a vertex v in Γ is permutation isomorphic to L. The maximum growth of $|G_v|$ as a function of $|V\Gamma|$ for locally-L pairs (Γ, G) is called the graph growth of L. In this talk, I will discuss a recent result showing that if a transitive permutation group L on a finite set Ω admits a proper block B such that the pointwise stabiliser of $\Omega \setminus B$ in L is non-trivial, then the graph growth of L is exponential. I will illustrate the impact of this new result by providing a survey on graph growth types of transitive permutation groups of low degree. This is joint work with Gabriel Verret.

Stefan Gyurki (Slovak University of Technology, Bratislava)

Time: 11:22–11:44

Small vertex transitive directed strongly regular graphs

In the talk we report about a systematic computer assisted search for small vertex transitive directed strongly regular graphs. The enumeration of such graphs was previously known up to order 20, here we extend the catalogue of such digraphs up to order 31. Our enumeration has been done using computer system GAP and we were essentially relying on the existing catalogue of small association schemes by Hanaki and Miyamoto.

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Jozef Širáň (Slovak University of Technology, Bratislava)

Time: 11:44–12:06

Orientably-regular embeddings of complete multigraphs

An embedding of a graph on an orientable surface is *orientably-regular* (or *rotary*, in an equivalent terminology) if the group of orientation-preserving automorphisms of the embedding is transitive (and hence regular) on incident vertex-edge pairs of the graph.

A classification of orientably-regular embeddings of complete graphs was obtained by L. D. James and G. A. Jones (1985), pointing out interesting connections to finite fields and Frobenius groups.

By a combination of methods from abstract and combinatorial group theory we extend the results of James and Jones to classification of orientably-regular embeddings of complete multigraphs with arbitrary edge-multiplicity.

Extension of results on orientably-regular embeddings of simple graphs to *multigraphs* in general is motivated by the fact that the latter appear to be prevalent for bounded genus. For example, by computational results of M. Conder, among all orientably-regular maps on surfaces up to genus 300 only about 11 per cent are such that both the map and its dual have a simple underlying graph.

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Marcel Jackson (La Trobe University)

Time: 12:06–12:28

Chromatic Algebras

Non-associative Relation Algebras (nRA) form the basic framework for the algebraic foundations of qualitative reasoning models. From a purely combinatorial perspective, the "chromatic algebras" are perhaps the simplest class to motivate, as qualitative representations correspond to a kind of Ramsey-theoretic condition on edge colourings of complete graphs: each 3-clique must avoid some forbidden combination of colours, but also, each non-forbidden 3-clique colouring must appear somewhere. In this talk we give an overview of some of the interesting problems in this area, and of the complete classification of which combinations of forbidden colour combinations allow for both constraints to be satisfied. As well

as touching on classical Ramsey theory, the talk will also touch on quasigroups, and finite linear spaces (in the sense of incidence geometries). This work is joint with Badriah al Juaid (La Trobe University and Taif University), Tomasz Kowalski (Jagiellonian University) and James Koussas (La Trobe University).
