

# TS8: Thematic Session: Inverse semigroups, restriction semigroups and related algebras

Monday 30 June, 14:00–16:00 • Room 106

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**Ruy Exel** (Universidade Federal de Santa Catarina, Florianópolis)

Time: 14:00–14:30

## Twisted Steinberg algebras, regular inclusions and induction

Given a field  $K$  and an ample (not necessarily Hausdorff) groupoid  $G$ , we define the concept of a *line bundle* over  $G$  inspired by the well known concept from the theory of  $C^*$ -algebras. If  $E$  is such a line bundle, we construct the associated *twisted Steinberg algebra* in terms of sections of  $E$ , which turns out to extend the original construction introduced independently by Steinberg in 2010, and by Clark, Farthing, Sims and Tomforde in a 2014 paper (originally announced in 2011). We also generalize (strictly, in the non-Hausdorff case) the 2023 construction of (cocycle) twisted Steinberg algebras of Armstrong, Clark, Courtney, Lin, McCormick and Ramagge. We then extend Steinberg's theory of induction of modules, not only to the twisted case, but to the much more general case of *regular inclusions* of algebras. Our main result shows that, under appropriate conditions, every irreducible module is induced by an irreducible module over a certain abstractly defined *isotropy algebra*. This is joint work with Misha Dokuchaev and Hector Pinedo, based on a recent paper dedicated to the memory of Fernando Abadie.

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**Ganna Kudryavtseva** (University of Ljubljana )

Time: 14:30–15:00

## Generalizing Steinberg algebras to algebras of ample categories

We generalize a result by Steinberg and show that the contracted semigroup algebra  $K_0S$  of a restriction semigroup  $S$  with a restriction zero over a commutative ring with unit  $K$  is isomorphic to the algebra  $K(\mathcal{CS})$  of the universal category  $\mathcal{CS}$  of  $S$ . This also extends the result of Stein on algebras of left restriction and Ehresmann semigroups satisfying certain finitary conditions. We relate  $K_0S$  with the algebra of the universal and tight Booleanizations of  $S$ . The latter Booleanizations are analogues of similar notions for inverse semigroups considered by Lawson and Lenz and are Boolean restriction semigroups which, as follows from recent work by the author extending the result by Cockett and Garner for the unital case, are related to Boolean inverse semigroups similarly to as ample categories are related to ample groupoids.

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**Mikhailo Dokuchaev** (Universidade de São Paulo)

Time: 15:00–15:30

## (Co)homology of skew product algebras by inverse monoid actions with application to Steinberg algebras

Given a unital action of an inverse monoid  $S$  on an algebra  $A$  and a bimodule  $M$  over the skew product algebra  $A *_\theta S$ , we construct a (co)homology Grothendieck spectral sequence which converges to the Hochschild (co)homology of  $A *_\theta S$  with values in  $M$ . The spectral sequence involves the (co)homology of  $S$  and the Hochschild (co)homology of  $A$ , when applied to unital Steinberg algebras over a field and also in the case of an  $E$ -unital  $S$ , whose action  $\theta$  on  $A$  is compatible. Moreover, in the case of a Steinberg algebra, the homology spectral sequence collapses on  $p$ -axis, resulting in an isomorphism of homology groups.

This is a joint work with Mykola Khrypchenko and Juan Jacobo Simón.

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**José Luis Vilca Rodríguez** (Universidade de São Paulo)

Time: 15:30–16:00

## Inverse semialgebras

We introduce the notion of an inverse semialgebra, both associative and Lie, and provide illustrative examples of these structures. Furthermore, we present several results, including, in particular, an analogue of the characterization of  $F$ -inverse semigroups as crossed products, adapted to the setting of Lie inverse semialgebras.

This is a part of a joint work with M. Dokuchaev and F. Johari.